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BULLETIN N° 228
ACADÉMIE EUROPEENNE INTERDISCIPLINAIRE DES SCIENCES

INTERDISCIPLINARY EUROPEAN ACADEMY OF SCIENCES

Lundi 1er octobre 2018 16h à l'Institut Henri Poincaré salle 01:

Conférence
"*Transport électronique quantique* "
par Gilles MONTAMBAUX
Directeur de Recherche de classe exceptionnelle CNRS
Laboratoire de Physique des Solides, Orsay
Professeur à l'École Polytechnique

Notre Prochaine séance aura lieu le lundi 5 novembre 2018
à 15h30
à l'Institut Henri Poincaré salle 01
11, rue Pierre et Marie Curie 75005 PARIS/Métro : RER Luxembourg
Elle aura pour thème

ASSEMBLEE GÉNÉRALE ANNUELLE de l'AEIS
Examen de candidature(s)

Académie Européenne Interdisciplinaire des Sciences
Siège Social : 5 rue Descartes 75005 Paris
http://www.science-inter.com

ACADÉMIE EUROPÉENNE INTERDISCIPLINAIRE DES SCIENCES
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octobre 2018

N°228

TABLE DES MATIERES

Prochaine séance : lundi 5 novembre 2018 IHP à 15h30 salle 01

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ACADEMIE EUROPEENNE INTERDISCIPLINAIRE DES SCIENCES
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5 rue Descartes 75005 PARIS

Séance du Lundi 1octobre 2018/Institut Henri Poincaré salle 01

La séance est ouverte à 16h, **sous la Présidence de Victor MASTRANGELO** et en la présence de nos Collègues Gilbert BELAUBRE, Jean-Louis BOBIN, Gilles COHEN-TANNOUDJI, Françoise DUTHEIL, Claude ELBAZ, Michel GONDRAN, Irène HERPE-LITWIN, Claude MAURY, Marie-Françoise PASSINI, Jacques PRINTZ, Jean SCHMETS, Alain STAHL, Jean-Paul TEYSSANDIER, Jean-Pierre TREUIL.

Sont excusés: François BEGON, Jean-Pierre BESSIS, Bruno BLONDEL, Michel CABANAC, Alain CARDON, Alain CORDIER , Juan-Carlos CHACHQUES, Eric CHENIN, Sylvie DERENNE, Ernesto DI MAURO, Jean Félix DURASTANTI, Vincent FLEURY, Jean-Pierre FRANÇOISE, Dominique LAMBERT, Gérard LEVY, Antoine LONG, Pierre MARCHAIS, Anastassios METAXAS, Jean-Jacques NIO, Alberto OLIVIERO, Edith PERRIER, Michel SPIRO, Jean VERDETTI

Etait présent en tant que visiteur Jean BERBINAU

I. Présentation du conférencier par notre Président Victor MASTRANGELO. Le Pr Gilles MONTAMBAUX nous a confié son CV:

Gilles Montambaux http://users.lps.u-psud.fr/montambaux

- Né le 25 janvier 1955
- Directeur de Recherche Classe Exceptionnelle au CNRS, Professeur à l'Ecole Polytechnique
- Laboratoire de Physique des Solides, Orsay, France

Parcours Professionnel
- 1974 Ecole Normale Supérieure de Cachan
- 1977 Agrégation de Physique
- 1980 Service National (2 ans) en coopération à l'Université de Tunis
- 1982 Chargé de Recherche au CNRS
- 1985 Thèse d'Etat à Orsay : Contribution à l'étude des conducteurs quasi-unidimensionnels sous champ magnétique
- 1986 Séjour post-doctoral aux Bell Laboratories (USA)
- 1992 Professeur chargé de cours à l'Ecole Polytechnique
- 1993 Directeur de Recherche CNRS
- 2008 Professeur à l'Ecole Polytechnique
- 2011 Directeur de Recherche Classe Exceptionnelle CNRS

Recherche

Thèmes de recherche :

- **Propriétés électroniques des systèmes mésoscopiques,**
- désordre, interactions et cohérence de phase en matière condensée,
- physique à basse dimension,
- supraconductivité,
- systèmes hybrides normal/supraconducteur,
- ferromagnétique/supraconducteur,
- graphène,
- physique des points de Dirac en matière condensée et dans les gaz d'atomes froids.

Environ 200 articles publiés dans des revues internationales. Environ 80 invitations dans les conférences internationales. Co-auteur de deux livres en français et en anglais sur la **Physique mésoscopique des électrons et des photons.**
Détails des activités sur http://users.lps.u-psud.fr/montambaux
Publications : sur http://users.lps.u-psud.fr/montambaux/publications.htm

Enseignement

Enseignement à l'Ecole Polytechnique : Physique Statistique, Mécanique Quantique, Physique des Solides, **Physique Mésoscopique**, voir http://users.lps.u-psud.fr/montambaux/X.html
Cours dispensés dans une dizaine d'écoles internationales.

Animation et principales responsabilités collectives

(Co-)Organisateur d'une vingtaine de conférences et écoles internationales (les Houches 1994 et 2004)
Sous-directeur du Laboratoire de Physique des Solides d'Orsay, de 2001 à 2004, puis 2012-2013.
Directeur du GDR Physique Quantique Mésoscopique de 2001 à 2009.
Membre du bureau de la section Matière Condensée de la Société Française de Physique, 2003-2008.
Editeur Scientifique de "European Physical Journal B", 1997-2004, des "Annales de Physique", 2000-2009, de "European Physical Journal Special Topics" à partir de 2010.
Membre nommé de la section 06 du Comité National, 2004-2008
Responsable de l'équipe "théoriciens" du Laboratoire de Physique des Solides d'Orsay, 2007-2015
Membre du conseil scientifique de l'Institut de Physique du CNRS, de 2010 à 2014
Directeur-adjoint de l'Ecole Doctorale « Physique en Île de France », 2012-2016

Encadrement

Encadrement de huit thèses

Distinctions

Médaille de bronze du CNRS 1986, Prix Anatole et Suzanne Abragam de l'Académie des Sciences 1992, Chevalier dans l'ordre des Palmes Académiques, Grand Prix Servant de l'Académie des Sciences 2017.

II. Conférence du Pr Gilles MONTAMBAUX :

Résumé de la conférence avec références bibliographiques:

Transport électronique quantique

Gilles Montambaux, Laboratoire de Physique des Solides, Université Paris-Sud

La miniaturisation des circuits électroniques et le développement des nanotechnologies ont permis de mettre en évidence de nouveaux effets quantiques qui régissent le transport électrique. Ces phénomènes apparaissent à une échelle intermédiaire entre l'échelle macroscopique de notre quotidien et l'échelle atomique, nanoscopique. C'est ce qu'on appelle le monde mésoscopique. La physique mésoscopique se développe au carrefour de problématiques à la fois appliquées et conceptuellement nouvelles. Le caractère quantique des électrons qui se comportent alors comme des ondes devient primordial. On décrira ici quelques-uns de ces effets nouveaux où le rôle combiné de la cohérence de phase et du désordre conduit à des effets subtils. Il devient délicat de séparer l'objet quantique à étudier et le monde macroscopique qui le mesure. Ces nouvelles propriétés ne sont pas nécessairement propres aux électrons mais se manifestent aussi dans la propagation d'autres ondes, lumineuses, micro-ondes, acoustiques, etc. Les analogies entre ces différents champs thématiques sont fécondes.

Un compte-rendu détaillé, voire **un enregistrement audio-vidéo** sera prochainement disponible sur le site de l'AEIS http://www.science-inter.com

Annonces

I. Notre président vous informe de la parution de l'ouvrage relatif à notre colloque de 2016 " ONDES , MATIÈRE ET UNIVERS" chez EDP sciences. **La version PDF est d'ores et déjà disponible en Open-Access sur le site d'EDP-Sciences :** https://www.edp-open.org/books-in-french#Ondes_matiere_et_Univers

Grâce à notre collègue Jean SCHMETS, cette information a été diffusée sur le site européen CORDIS avec la référence https://cordis.europa.eu/news/rcn/130072_en.html

II. **La version papier est également disponible:**

Coût de l'ouvrage papier : 40€

Mode d'acquisition : adresser à notre Trésorière Edith PERRIER un chèque ou un virement correspondant aux nombres d'exemplaires souhaités.

Coordonnées Edith PERRIER:

PERRIER Edith	Bonneval 19120 PUY D'ARNAC	edith.perrier@ird.fr	01 48 02 59 69	06 83 05 72 04

(*Si nécessaire elle vous communiquera le RIB de virement*)

Dès que notre Trésorière aura perçu les fonds, les ouvrages vous seront remis lors de la prochaine séance de l'AEIS.

III. **Quelques ouvrages papiers relatifs au colloque de 2014 " Systèmes stellaires et planétaires- Conditions d'apparition de la Vie"** - restent encore disponibles:

- Prix de l'ouvrage :25€ .
- Pour toute commande s'adresser à Edith Perrier (voir coordonnées ci-dessus):

L'ouvrage cité ci-dessus est accessible gratuitement (open access) sur le site d'edp sciences:

http://www.edp-open.org/images/stories/books/fulldl/Formation-des-systemes-stellaires-et-planetaires.pdf

Documents

Nous vous proposons:

I.

p.06 la diffusion sur le site cordis https://cordis.europa.eu/news/rcn/130072_en.html de l'annonce de la parution chez EDP Sciences en open access de l'ouvrage relatif au colloque de 2016 "Ondes Matière et Univers"

II.

p. 08 Le résumé d'un article de notre collègue Claude ELBAZ publié sur le site http://www.ccsenet.org/journal/index.php/apr/article/view/0/36945 de la revue canadienne "Applied Physics Research;Vol l0 , N°5 ; 2018 intitulé " Incompleteness of General Relativity Regarding Einstein's Program".

III. Pour compléter la conférence du Pr Gilles MONTAMBAUX relative au futur colloque "Les Signatures quantiques des états mésocopiques" :

p. 11 un cours donné à Cargèse en avril 2018 par le Pr MONTAMBAUX intitulé "Quantum Transport in 2D"

Book in French and English: Waves Matter and Universe, General Relativity, Quantum Physics and ApplicationsEditor: edp SciencesCollection: AEIS

Contributed by: **Interdisciplinary European Academy of Sciences • Académie Européenne Interdisciplinaire des Sciences**

Title: Ondes Matière et Univers • Relativité générale, Physique quantique et Applications

Contributed by: Interdisciplinary European Academy of Sciences • Académie Européenne Interdisciplinaire des Sciences

Presentation:

Integrated in the multidisciplinary view of our Academy, this book shows the scientific advances accomplished since Einstein's first works at the beginning of the XXth century in the main two fields of modern Physics, which are General Relativity and Quantum Physics. It is aimed at collecting the most significant scientific results and observations from the end of the XXth century and the beginning of the 3rd Millennium. Despite the present problems of attempts to conciliate them in order to build a unified theory, these two theories show to be very relevant in each of their relative fields of validity. They offer an unimaginably rich view of the world and the universe.

This book comprises four parts:

1) Relativity, Waves of the Universe

2) Particle-Wave Duality in Quantum Physics

3) Waves, Matter and Quantification

4) A new Scientific Revolution on the horizon?

Contributors:

A. ASPECT (Institut d'Optique Graduate School, Université Paris-Saclay, École Polytechnique), F. BALIBAR (Université Paris Diderot Paris 7), F. BOUCHET (Institut d'Astrophysique de Paris, CNRS & Sorbonne Université-UPMC), J. BRICMONT (UCL), C. COHEN-TANNOUDJI (Nobel Prize 1997, Laboratoire Kastler Brossel, CNRS, ENS-PSL Research University, Collège de France), J. DALIBARD (Collège de France and Laboratoire Kastler Brossel, CNRS, ENS-PSL Research University, UPMC-Sorbonne Universités), B. DIU (Université Paris Diderot Paris 7), S. HAROCHE (Nobel Prize 2012, Laboratoire Kastler Brossel, CNRS, ENS-PSL Research University, Collège de France), P. HELLO (Laboratoire de l'Accélérateur Linéaire, Orsay, CNRS, IN2P3 and Université Paris Sud), F. LALOË (Laboratoire Kastler Brossel, ENS-UMPC), D. LAMBERT (Université de Namur), J.M. RAIMOND (Laboratoire Kastler Brossel, CNRS, ENS-PSL Research University, UPMC-Sorbonne Université), C. SALOMON (Laboratoire Kastler Brossel ENS-PSL Research University; CNRS, UPMC-Sorbonne Université, Collège de France), M. SERRES (Académie Française), S. SPEZIALE (Centre de Physique Théorique, CNRS, Université d'Aix Marseille & Université de Toulon), G. VENEZIANO (CERN), J. ZINN-JUSTIN (CEA Centre de Paris-Saclay), J.P. TREUIL (AEIS)

Reference: ISBN: 978-2-7598-2264-5 July 2018, 534 pp

Information:

The Interdisciplinary European Academy of Sciences announces that this book can be downloaded free of charge at the following address:

https://www.edp-open.org/books-in-french#Ondes_matiere_et_Univers

Contributor

Organisation	Interdisciplinary European Academy of Sciences - Académie Européenne Interdisciplinaire des Sciences Rue Descartes 5 F-75005 Paris France Website
Contact	Professor Dr. Jean SCHMETS Tel.: +3242527873 E-mail
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Related information

Programmes	H2020

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Subjects

Scientific Research

Keywords

General Relativity, gravitational wave, Cosmology-Planck Satellite, Quantum mechanics, Quantum Physics, gas atoms cooled, EPRB paradox, Bell's inequality, entangled photons, The photon box of Einstein and Bohr, microwave Cavity Quantum Electrodynamics, Renormalization and renormalization group, string theory, String cosmology, Loop quantum gravity, Black holes **Last updated on** 2018-10-03

Retrieved on 2018-10-22

Permalink : https://cordis.europa.eu/news/rcn/130072_en.html

Incompleteness of General Relativity Regarding Einstein's Program

http://www.ccsenet.org/journal/index.php/apr/article/view/0/36945

- Claude Elbaz

Abstract

The detection of gravitational waves substantiates the undeniable achievement of general relativity theory by increasing its theoretical and experimental accuracy. One century after predicting it has set again Einstein's works at the front of research. Absence of quantum particle associated to gravitation emphasizes that general relativity theory remains not included in the standard model of physics. Then Einstein's disagreement about it incompleteness regarding wave-particle and matter-field becomes actualized. In order to circumvent these difficulties he privileged field, rather than matter for universe description in his program. In consequence a scalar field $e(r_0,t_0)$ propagating at speed of light c yields matter from standing waves moving at speed strictly inferior to c, and interactions from progressive waves. Electromagnetic interactions derive from local variations of frequencies, and gravitation from local variations of speed of light. A space-like amplitude functions $u_0(k_0r_0)$ supplements fundamental time-like functions of classical and quantum mechanics. It tends toward Dirac's distribution Delta (r_0) in geometrical optics approximation conditions, when frequencies are infinitely high, and then hidden.

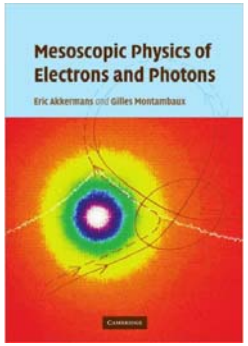
More generally, it allows theoretical economies by deriving energy-momentum conservation laws, and least action law. Quantum domain corresponds to wave optics approximation conditions. Variations of frequencies give rise to an adiabatic constant, formally identical with Planck's constant, leading to first quantification for electromagnetic interactions and to second quantification for matter.

GRAPHENE & CO, Cargèse April 2-13, 2018

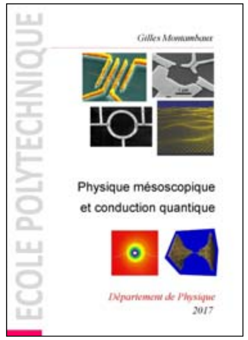
Quantum transport in 2D

Gilles Montambaux, Université Paris-Sud, Orsay, France

users.lps.u-psud.fr/montambaux



*Quantum transport,
weak-localization, UCF, etc.*



*Introductions à Landauer-Büttiker,
transport quantique, graphène :
Poly en français accesssible sur*

users.lps.u-psud.fr/montambaux

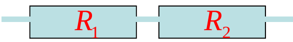
Publications on motion and merging of Dirac cones in graphene and artificial graphenes

Quantum transport = Mesoscopic physics = Phase coherence

Breakdown of classical laws of electronic transport

$$R \neq R_1 + R_2$$

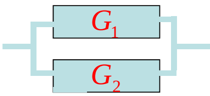
$$R \neq \rho \frac{L}{S}$$



$$G = \frac{1}{R}$$

$$G \neq G_1 + G_2$$

$$G \neq \sigma \frac{S}{L}$$



cf. Two path interferometer…

Outline

I - From classical transport to quantum transport

Disorder and phase coherence, the important length scales

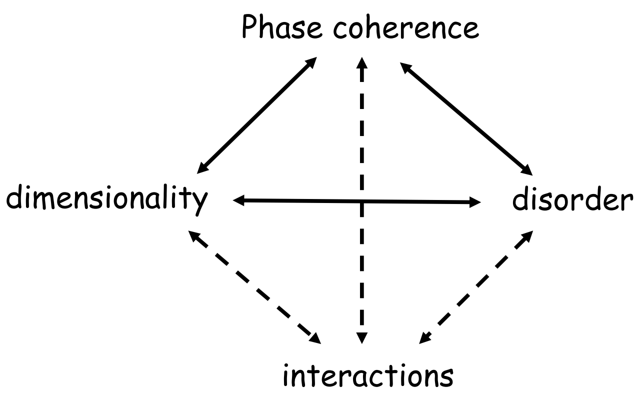
Different regimes of transport
- Ballistic classical (Sharvin)
- Ballistic quantum (quantization of conductance)
- Diffusive classical (Ohm-Drude)
- Diffusive quantum (weak-localization , UCF)

What is specific in 2D ? Disorder effect and weak-localization

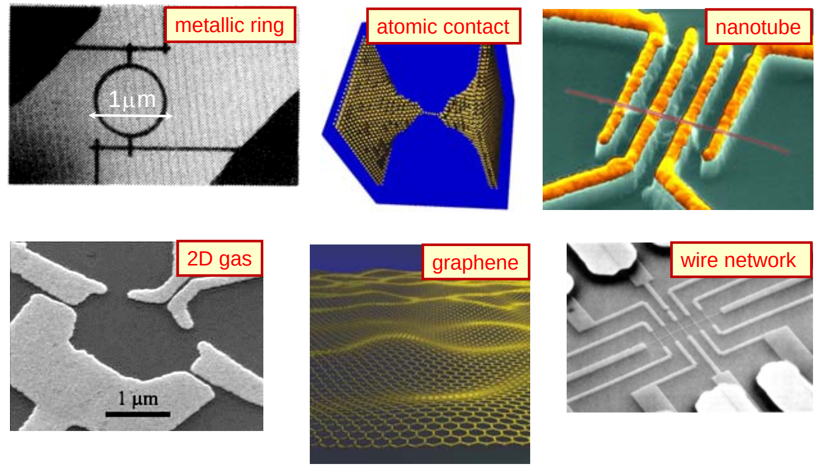
II - Landauer-Büttiker formalism of quantum transport
- Two terminal vs four terminal measurements
- Multiterminal formalism
- Application to QHE

III - Dirac matter, graphene and other materials (BN, bilayer, phosphorene)
- Engineering of Dirac points

The mesoscopic triangle



Quantum transport : what is conductance?



Landauer-Büttiker : conductance = transmission

Outline

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Disorder and phase coherence, the important length scales

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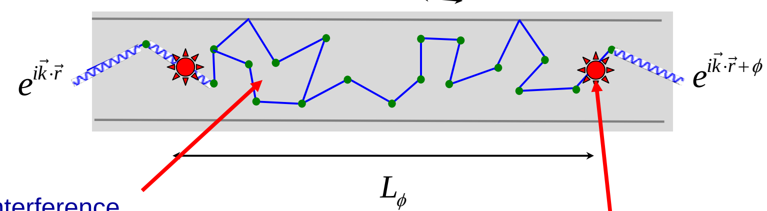
- Diffusive classical (Ohm-Drude)
- Diffusive quantum (weak-localization , UCF)

What is specific in 2D ? Disorder effect and weak-localization

Length scales

l_e Mean free path : distance between elastic collisions

$\lambda_F \ll l_e$ $l_e = v_F \tau_e$ elastic collision time



interference

Elastic collisions do not break phase coherence

Interaction with an external degree of freedom (phonons, electrons, spin impurities… breaks phase coherence

$L_\phi(T)$ Phase coherence length $L_\phi = \sqrt{D\tau_\varphi}$

Correspondance: time ←→ Length in the diffusive regime

phase coherence length $L_\phi^2 = D\,\tau_\phi$ phase coherence time

System size $L^2 = D\,\tau_D$

Thouless time

diffusion time

characteristic traversal time

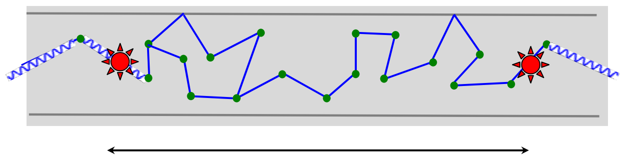
More generally any (cut-off) time will be related to a characteristic length

$$L_c^2 = D\,\tau_c$$

Length scales

l_e Mean free path : distance between elastic collisions

$\lambda_F \ll l_e$ $\quad l_e = v_F \tau_e$



L_ϕ

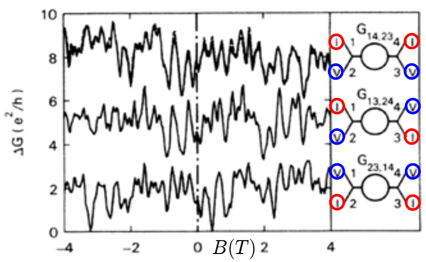
Weak-disorder (diffusive) mesoscopic regime $\lambda_F \ll l_e \ll L < L_\phi$

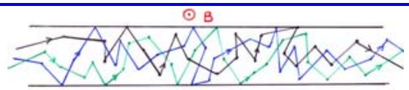
Ballistic regime $l_e \to \infty$

Strongly (Anderson) disordered regime $l_e < \lambda_F \ll L < L_\phi$

Reproducibles conductance fluctuations

$l_e \ll L < L_\phi$





~~$G = \frac{I}{V}$~~ → $G_{ij,kl} = \frac{I_{ij}}{V_{kl}}$

The conductance does not depend only on the system to be studied, but also to its connection to the external world

$\delta G \sim \frac{e^2}{h}$

One measures a conductance and not a conductivity

The Drude conductivity is an average property, valid if $L_\phi < L$

→ Beyond the average : interferences, fluctuations …

Disorder and phase coherence are independant notions

Disorder → complex interference pattern

This interference pattern vanishes when phase coherence is lost

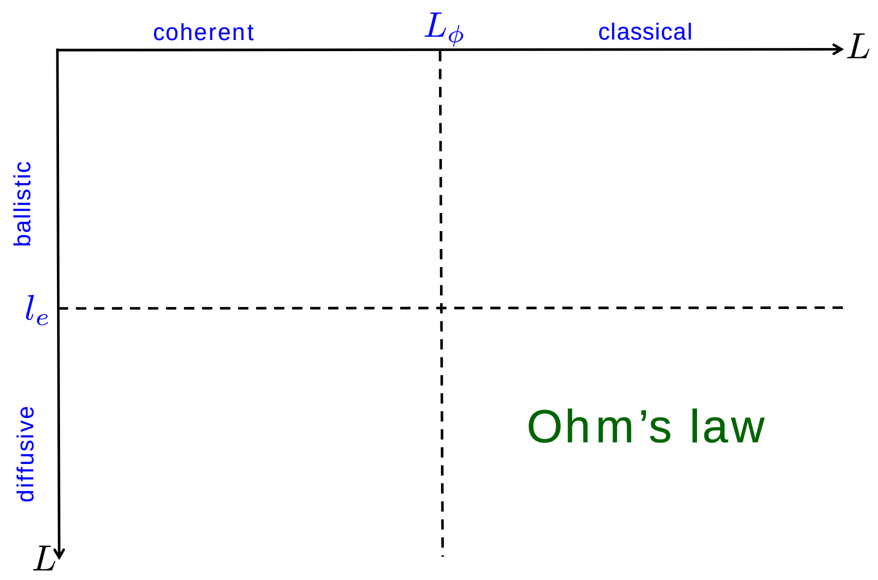
Phase coherence is limited by the coupling to other degrees of freedom :

phonons
magnetic impurities
other electrons

$\longrightarrow L_\phi$

Disorder does not kill the interference pattern, makes it more complex

What about disorder average ?



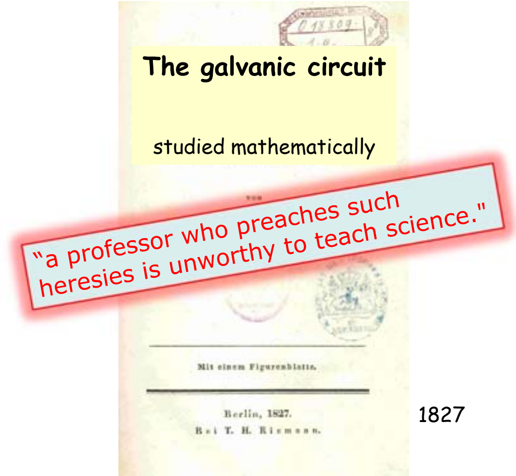
$k_F l_e \gg 1$

Ohm's law

1789-1854

$$I = GV$$

$$G = \sigma \frac{S}{L}$$

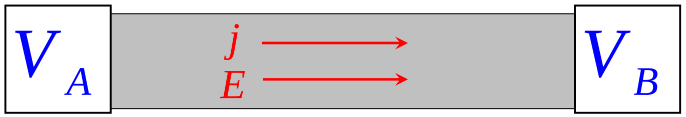


1827

G conductance, σ conductivity

$$G = 1/R$$

Ohm's law



$$\vec{j} = \sigma \vec{E}$$

$$I = GV$$

$$I = jS = \sigma E S$$

$$V = V_A - V_B = E L$$

$$G = \sigma \frac{S}{L}$$

G conductance, σ conductivity

Ohm's law

$$\sigma = \frac{ne^2\tau}{m}$$

Drude-Sommerfeld formula
1900

$$G = \sigma \frac{S}{L}$$

$$\sigma = e^2 D \rho_0$$

Einstein formula

D diffusion coefficient

ρ_0 DOS at the Fermi level

Validity

Diffusive regime $L \gg l_e$

No quantum effects $L > L_\phi$

Brief reminder on density of states

$$\frac{1}{V}\sum_j \varphi(\epsilon_j) = \int \rho(\epsilon)\varphi(\epsilon)d\epsilon$$

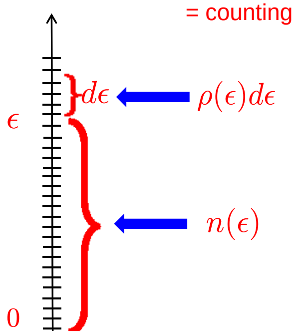
Number of states at a given energy : $\rho(\epsilon) = \frac{1}{V}\sum_j \delta(\epsilon - \epsilon_j)$

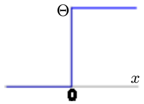
Number of states in an energy window $[\epsilon, \epsilon + d\epsilon]$: $\rho(\epsilon)d\epsilon$

It is very useful to define

= number of states of energy smaller than ϵ
= integrated DOS
= counting function

$$n(\epsilon) = \frac{1}{V}\sum_j \Theta(\epsilon - \epsilon_j)$$





$$\rho(\epsilon) = \frac{dn(\epsilon)}{d\epsilon}$$

by definition :
$n = n(\epsilon_F)$

Brief reminder on density of states

$$n(\epsilon) = \frac{1}{V}\sum_{\vec{k}} \Theta(\epsilon - \epsilon_{\vec{k}})$$

Continuum limit : $n(\epsilon) = \frac{1}{(2\pi)^d}\int_{\epsilon_{\vec{k}}<\epsilon} d^3\vec{k}$

$$n(\epsilon) = \frac{\text{Volume of } \vec{k} \text{ space } |\epsilon_{\vec{k}} < \epsilon}{(2\pi)^d}$$

If $\epsilon(\vec{k}) = \epsilon(|\vec{k}|)$

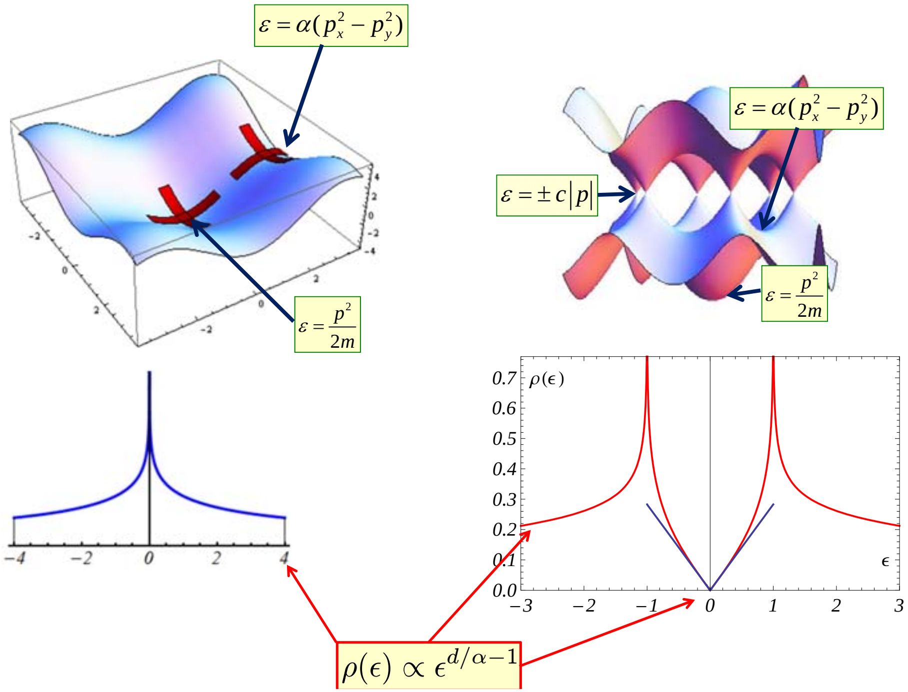
$$\rho(\epsilon) = \frac{dn}{d\epsilon}$$

$$n(\epsilon) = \frac{A_d}{(2\pi)^d}k(\epsilon)^d$$

$\epsilon(\vec{k}) \propto k^\alpha$

$$\rho(\epsilon) \propto \epsilon^{d/\alpha - 1}$$

$$\rho_0 = \rho(\epsilon_F) = \frac{dA_d}{\lambda_F^{d-1}hv_F}$$



What can we learn form classical transport ? back to Ohm's law

Ohm's law $G_{diff} = \sigma\frac{S}{L} = \sigma\frac{W^{d-1}}{L}$

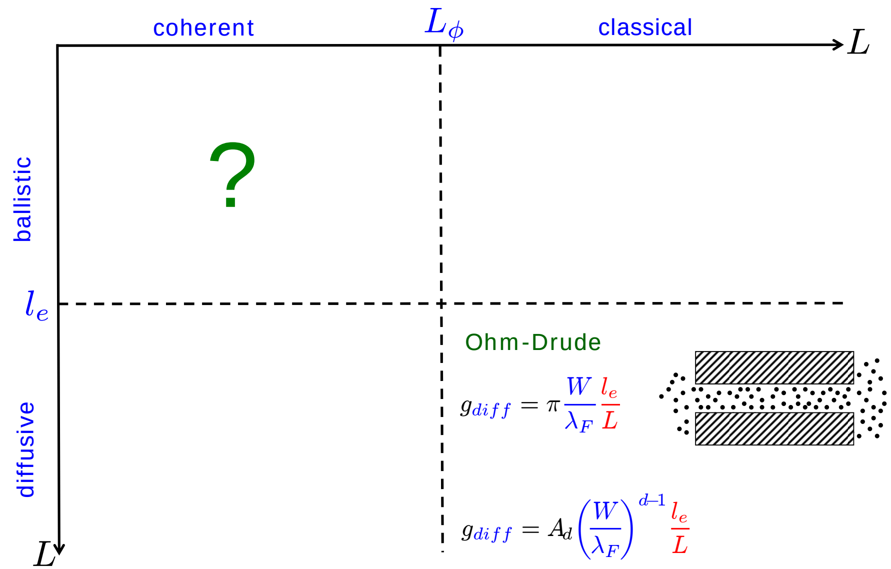
Conductivity $\sigma = e^2 D\rho_0$

Diffusion coefficient $D = \frac{v_F^2\tau_e}{d} = \frac{v_F l_e}{d}$

$$G_{diff} = e^2\rho_0 S\frac{v_F}{d}\frac{l_e}{L}$$

Density of states $\rho_0 = \rho(\epsilon_F) = \frac{2dA_d}{\lambda_F^{d-1}hv_F}$

$$G_{diff} = 2\frac{e^2}{h}A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$$



$$g_{diff} = \pi \frac{W}{\lambda_F} \frac{l_e}{L}$$

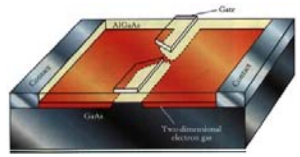
$$g_{diff} = A_d \left(\frac{W}{\lambda_F}\right)^{d-1} \frac{l_e}{L}$$

$k_F l_e \gg 1$

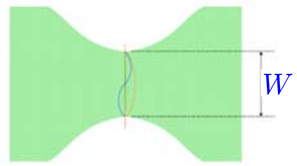
in units of $2\frac{e^2}{h}$

Conductance of a coherent ballistic system

$$G_q = 2\frac{e^2}{h} M = 2\frac{e^2}{h} \mathrm{Int} \frac{2W}{\lambda_F}$$

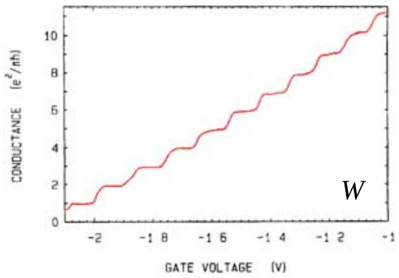


Quantum point contact **QPC**



wave guide

M transverse 'channels' 'modes'



Van Wees et al. PRL 1988; Wharam et al. J. Phys. ***C*** *1988*

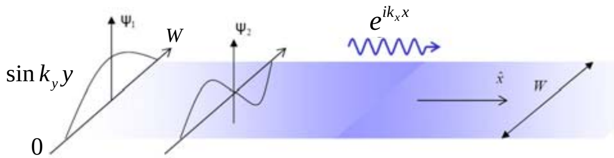
$2\frac{e^2}{h}$ per mode ...

see tomorrow's lecture on Landauer formula

Number of transverse channels

quantized modes

$$k_y = \frac{n_y \pi}{W}, \quad n_y > 0$$



since $k < k_F$ ⇒ $\frac{n_y \pi}{W} < k_F$

Number of transverse modes, channels

⇒ $$M = \mathrm{Int}\left(\frac{k_F W}{\pi}\right) = \mathrm{Int}\left(\frac{2W}{\lambda_F}\right)$$

Number of transverse channels

d=2 $$M = \mathrm{Int}\frac{k_F W}{\pi} = \mathrm{Int}\, 2\frac{W}{\lambda_F}$$

d=3 $$M = \mathrm{Int}\frac{\pi}{4}\left(\frac{k_F W}{\pi}\right)^2 = \mathrm{Int}\,\pi\frac{W^2}{\lambda_F^2}$$

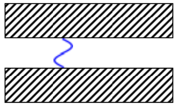
(d) $$M = \mathrm{Int}\frac{A_{d-1}}{2^{d-1}}\left(\frac{k_F W}{\pi}\right)^{d-1} = \mathrm{Int}\, A^{d-1}\frac{W^{d-1}}{\lambda_F^{d-1}}$$

coherent L_ϕ classical L

ballistic

Quantized conductance

$g_q = \text{Int}\, 2\frac{W}{\lambda_F}$



?

$g_q = \text{Int}\, A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

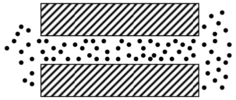
l_e

diffusive

M transverse 'channels' 'modes'

Ohm-Drude

$g_{diff} = \pi\frac{W}{\lambda_F}\frac{l_e}{L}$



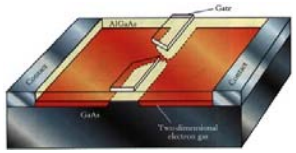
$g_{diff} = A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$

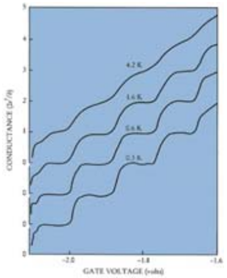
L

$k_F l_e \gg 1$

Conductance of a classical ballistic system

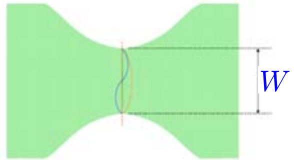
$$G_q = 2\frac{e^2}{h}M = 2\frac{e^2}{h}\text{Int}\frac{2W}{\lambda_F}$$



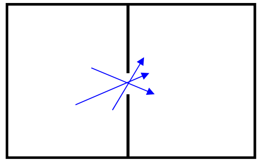


classical

coherent

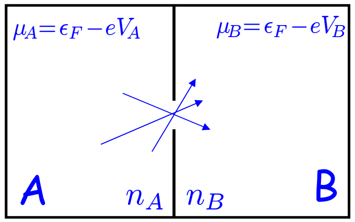


wave guide



What do we get for classical particles (fermions) ?

Sharvin (1965)



$S = W^{d-1}$

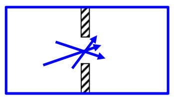
Number of particules of velocity v through the area S with incident angle θ during dt

→ Particle current $I_p = (n_A - n_B) S \langle v_x \rangle_+$

→ Electric current $I = (V_A - V_B) e^2 \rho S \langle v_x \rangle_+$

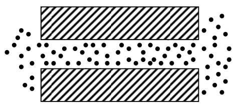
$G_{bal} = e^2 \rho_0 \langle v_x \rangle_+ S$

« Sharvin conductance »



$G_{diff} = e^2 \rho_0 \frac{v_F}{d} S \frac{l_e}{L}$

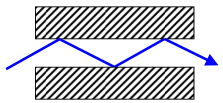
« Drude conductance »



Ballistic vs. diffusive incoherent

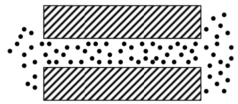
$G_{bal} = e^2 \rho_0 \langle v_x \rangle_+ S$

« Sharvin conductance »



$G_{diff} = e^2 \rho_0 \frac{v_F}{d} S \frac{l_e}{L}$

« Drude-Ohm conductance »



$$\rho_0 = \frac{2dA_d}{\lambda_F^{d-1} h v_F}$$

$$\langle v_x \rangle_+ = v_F \frac{A_{d-1}}{dA_d}$$

$G_{bal} = 2\frac{e^2}{h}A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

Sharvin

$G_{diff} = 2\frac{e^2}{h}A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$

Drude

If ballistic coherent :

$G_q = 2\frac{e^2}{h}A_{d-1}\text{Int}\left(\frac{W}{\lambda_F}\right)^{d-1}$

If diffusive coherent, quantum corrections

Weak-localization

coherent L_ϕ classical $\to L$

ballistic

Quantized conductance

$g_q = \text{Int}\, 2\frac{W}{\lambda_F}$

$g_q = \text{Int}\, A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

Sharvin

$g_{bal} = 2\frac{W}{\lambda_F}$

$g_{bal} = A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

l_e

diffusive

M transverse 'channels' 'modes'

Ohm-Drude

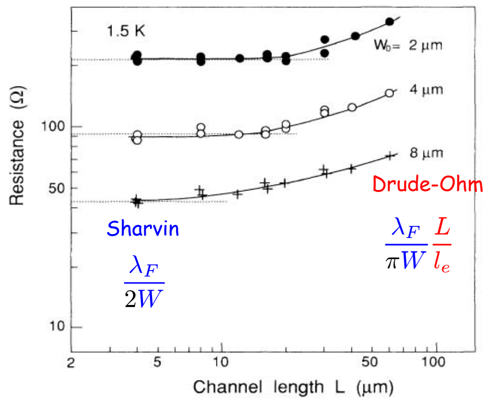
$g_{diff} = \pi\frac{W}{\lambda_F}\frac{l_e}{L}$

$g_{diff} = A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$

L

$k_F l_e \gg 1$

Ballistic $L < l_e$ $L > l_e$ Diffusive in units $\frac{h}{2e^2}$



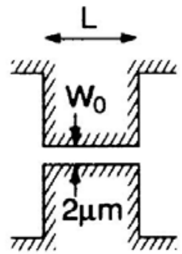


FIG. 2. Two-terminal resistances measured for the channels 2, 4, and 8 µm wide.

S. Tarucha et al., Sharvin resistance and its breakdown observed in long ballistic channels Phys. Rev. B 47, 4064 (1993)

coherent L_ϕ classical $\to L$

ballistic

Quantized conductance

$g_q = \text{Int}\, 2\frac{W}{\lambda_F}$

$g_q = \text{Int}\, A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

Sharvin

$g_{bal} = 2\frac{W}{\lambda_F}$

$g_{bal} = A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

l_e

diffusive

Weak-localization corrections

$g = g_{diff} + \Delta g$

Ohm-Drude

$g_{diff} = \pi\frac{W}{\lambda_F}\frac{l_e}{L}$

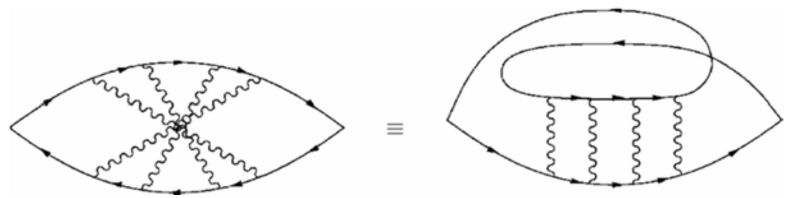
$g_{diff} = A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$

L

$k_F l_e \gg 1$

Quantum corrections in disordered media

Weak-localization

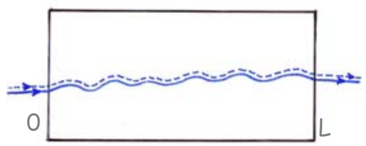


Conductance, transmission and probability

Conductance = transmission

Transmission through a disordered system= probability to cross the system

$$\overline{G} \propto P(0,L)$$



classical transport

Diffusion probability, microscopic approach

$P(r,r',t)$ probability to find a particule at *r'* , if it has been injected at *r*

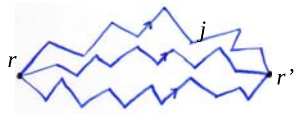
Quantum amplitude

Cf. Young's slits

$$G(r,r') = \sum_j A_j(r,r')$$

$$A_j(r,r') = |A_j(r,r')|\, e^{i\varphi_j(r,r')}$$

$$\varphi_j(r,r') = \frac{1}{\hbar}\int_r^{r'} p.dl$$



The probabiliy is the modulus square of the amplitude :

$$P(r,r') \sim \overline{|G(r,r')|^2} = \overline{\left|\sum_j A_j(r,r')\right|^2}$$

Disorder average

Diffusion probability, microscopic approach

Two contributions

Disorder average

$$P(r,r') = \overline{\sum_j |A_j(r,r')|^2} + \overline{\sum_{j\neq j'} A_j(r,r')A^*_{j'}(r,r')}$$

Classical term

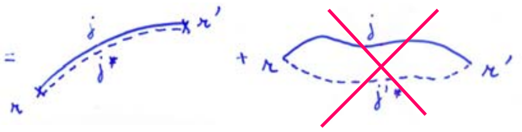
Interference term

Quantum effects

Classical transport : only paired trajectories A_j A_j contribute
If the trajectories are different, the amplitudes A_j et $A_{j'}$ are different

⟹ uncorrelated phases
⟹ In average, the interference term disappears

$P(r,r')$ =

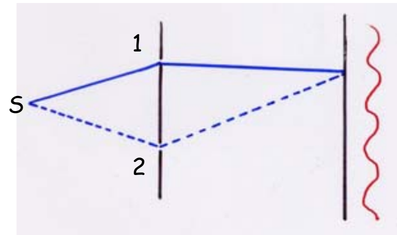


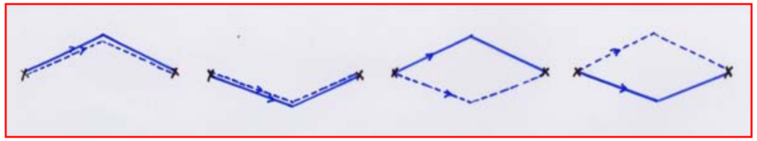
$P_{cl}(r,r')$

Example : Young slits

$$I = I_{cl} + I_{\text{int}}$$

$$I = |A_1 + A_2|^2$$

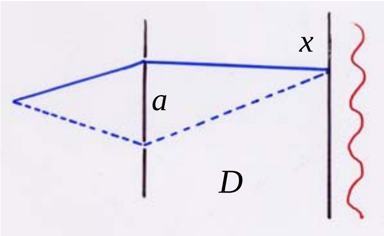




$$I = |A_1|^2 + |A_2|^2 + A_1\, A_2^* + A_2\, A_1^*$$

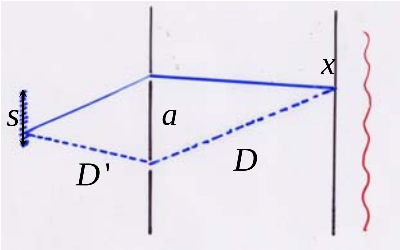
Example : Young slits

$$I = I_{cl} + I_{int}$$



$$I = I_{cl}\left(1+\cos\frac{kax}{D}\right)$$

$$I = I_{cl} + I_{int}$$



$$I = I_{cl}\left(1+\frac{\sin\frac{kas}{D'}}{\frac{kas}{D'}}\cos\frac{kax}{D}\right)$$

« disorder average »

$$\langle I_{int}\rangle = 0$$

Diffusion probability, microscopic approach

Two contributions

$$P(r,r') = \overline{\sum_j |A_j(r,r')|^2} + \overline{\sum_{j\neq j'} A_j(r,r')A^*_{j'}(r,r')}$$

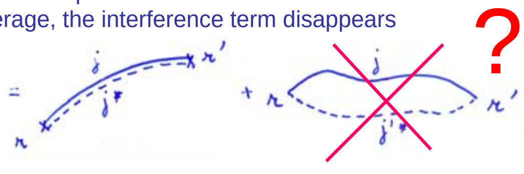
Classical term — Interference term

⟶ Quantum effects

Classical transport : only paired trajectories A_j A_j contribute
If the trajectories are different, the amplitudes A_j et $A_{j'}$ are different

⟹ uncorrelated phases
⟹ In average, the interference term disappears

$P(r,r')$ =

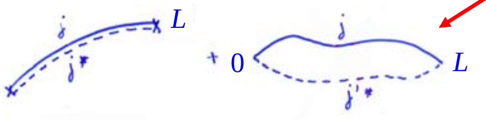


$P_{cl}(r,r')$

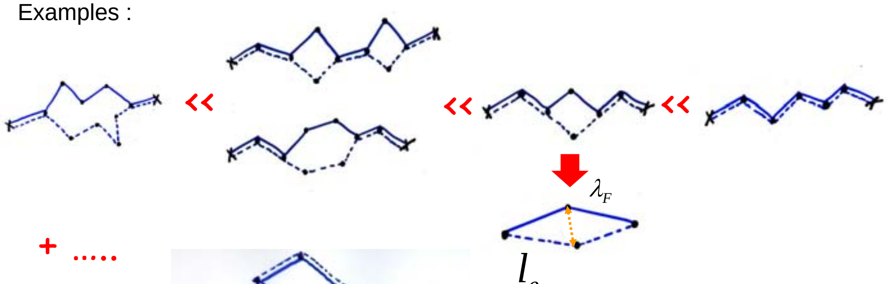
Quantum corrections

$$P(0,L) = \overline{\sum_j |A_j(0,L)|^2} + \overline{\sum_{j\neq j'} A_j(0,L)A^*_{j'}(0,L)}$$

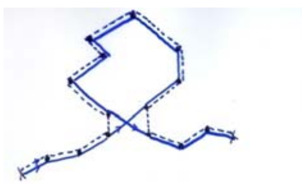
$P(0,L)$ =



Examples :



+ .....



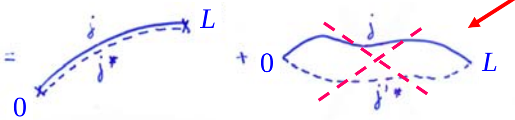
Quantum cross-section

λ_F^{d-1}

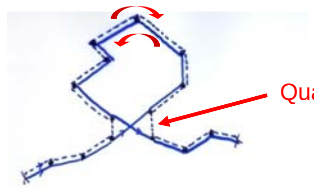
Quantum corrections

$$P(0,L) = \overline{\sum_j |A_j(0,L)|^2} + \overline{\sum_{j\neq j'} A_j(0,L)A^*_{j'}(0,L)}$$

$P(0,L)$ =



When performing disorder average, most contributions cancel, except when paired trajectories are very close to each other.
The remaining contribution corresponds to pairs of TR trajectories with a crossing.



Quantum crossing = Hikami box

At the crossing of trajectories, there is dephasing. When ageraging over the positions of impurities, one can show that :

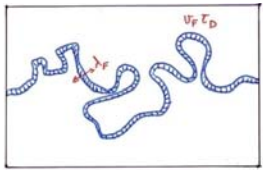
the relative amplitude of this contribution is of order of $\frac{1}{g}$ $\propto \lambda_F^{d-1}$

Evaluation of quantum corrections

The relative correction is proportionnal to the ratio :

$$\frac{\text{Volume of the trajectory explored during the diffusion through the sample}}{\text{Volume of the system}}$$

$$\boxed{\frac{\Delta G}{G} = \frac{\Delta P}{P} \propto \frac{\lambda_F^{d-1} v_F \tau_D}{L^d} \sim \frac{1}{g} \quad !!!}$$



Dimensionless conductance $G = g\,\frac{2e^2}{h}$

Coherent effects and quantum crossings

Quantum corrections are of relative order $\frac{1}{g}$

Classical transport $G_{cl} = g\,\frac{2e^2}{h}$

Quantum effects are of order $G_{cl} \times \frac{1}{g} \sim \boxed{\frac{e^2}{h}}$

Weak-localization corrections

Aharonov-Bohm (or Sharvin-Sharvin) oscillations

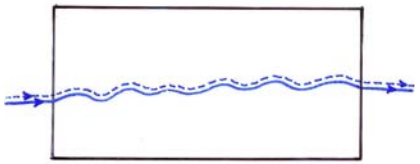
Universal conductance fluctuations

In a good metal ($g \gg 1$), quantum effects are small

Weak localization

Classical conductance

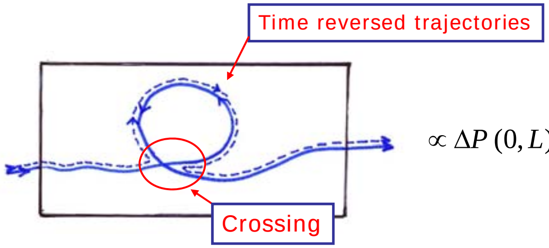
G_{cl}



$\propto P_{cl}(0,L)$

Quantum correction ⟹ **One crossing** ⟹ **One loop**

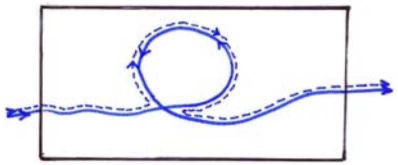
$$\boxed{\Delta G \sim -\frac{2e^2}{h}\langle P_{\text{int}}(t)\rangle}$$



$\propto \Delta P(0,L)$

$P_{\text{int}}(t)$ = distribution of number of loops with time t = <u>return probabililty</u>

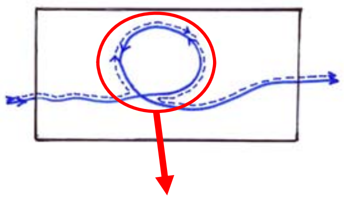
Weak-Localization



Loops and return probability
Weak-localisation in dimension d
Magnetic field, phase coherence

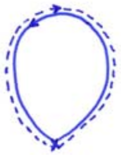
Weak localization : how to calculate $P_{\text{int}}(t)$?

$$\Delta G \sim -\frac{2e^2}{h}\left\langle P_{\text{int}}(t)\right\rangle$$



$P_{\text{int}}(t)$

Cooperon



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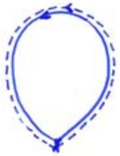


$P_{cl}(t)$

Diffuson

Interference term

Classical return probability

$$P_{\text{int}}(r,r,t) = P_{cl}(r,r,t)$$

If time reversal invariance

Classical probability, diffusion equation

$P_{cl}(r,r')$ is solution of a classical diffusion equation:

$$\left(\frac{\partial}{\partial t} - D\Delta\right) P_{cl}(r,r',t) = \delta(r-r')\delta(t)$$

Solution in free space in d dimensions: $$P_{cl}(R,t) = \frac{1}{(4\pi D t)^{d/2}} e^{-\frac{R^2}{4Dt}}$$

An important result, the return probability: $$P(r,r,t) = \frac{1}{(4\pi D t)^{d/2}}$$

volume explored after time t

Weak localization : cut-offs

$$\Delta g \sim -\left\langle P_{\text{int}}(t)\right\rangle = -\int_{\tau_e}^{\tau_c} P_{\text{int}}(t)\frac{dt}{\tau_D} \qquad P(t) = \frac{V}{(4\pi D t)^{d/2}}$$

Lower cut-off τ_e elastic collision time

Upper cut-off

time spent in the sample $\tau_D = \frac{L^2}{D}$

phase coherence time $\tau_\phi = \frac{L_\phi^2}{D}$

$$\tau_c = \min(\tau_D, \tau_\phi)$$

The return probability $P(t)$ increases for small d

Coherent effets are more important in low dimension

Weak localization : dependence on dimensionality

$$\Delta g \sim -\left\langle P_{\text{int}}(t)\right\rangle = -\int_{\tau_e}^{\tau_c} P_{\text{int}}(t)\frac{dt}{\tau_D} \qquad P(t) = \frac{V}{(4\pi D t)^{d/2}}$$

$$\int_{\tau_e}^{\tau_\phi} \frac{dt}{t^{d/2}} \rightarrow$$

- $\sqrt{\tau_\phi} - \sqrt{\tau_e}$ $\quad d=1 \quad (quasi-1D)$
- $\ln\frac{\tau_\phi}{\tau_e}$ $\quad d=2$
- $\frac{1}{\sqrt{\tau_e}} - \frac{1}{\sqrt{\tau_\phi}}$ $\quad d=3$

$$L_\phi^2 = D\tau_\phi$$

Weak localization : dependence on dimensionality

$d=1 \quad \Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L}$

$d=2 \quad \Delta g = -\frac{1}{\pi}\ln\frac{L_\phi(T)}{l_e} \qquad g = \frac{k_F l_e}{2}$

$d=3 \quad \Delta g = -\frac{1}{2\pi}\frac{L}{l_e} \qquad g = \frac{k_F^2 l_e L}{3\pi}$

Correction more important for small d because return probability is enhanced

$$g = A_d \left(\frac{k_F W}{2\pi}\right)^{d-1}\frac{l_e}{L} \quad \sim L^{d-2}$$

Weak localization : dependence on dimensionality

$d=1 \quad \Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L} \qquad \frac{\Delta g}{g} \sim \frac{L_\phi}{M\, l_e}$

$d=2 \quad \Delta g = -\frac{1}{\pi}\ln\frac{L_\phi(T)}{l_e} \qquad g = \frac{k_F l_e}{2} \qquad \frac{\Delta g}{g} = \frac{2}{\pi}\frac{\ln(L_\phi / l_e)}{k_F l_e}$

$d=3 \quad \Delta g = -\frac{1}{2\pi}\frac{L}{l_e} \qquad g = \frac{k_F^2 l_e L}{3\pi} \qquad \frac{\Delta g}{g} \propto \frac{1}{k_F^2 l_e^2}$

$\frac{\Delta g}{g} \sim 1$ defines a new length scale at which perturbation breaks down

Localization length : $\xi_{1D} \sim M\, l_e$

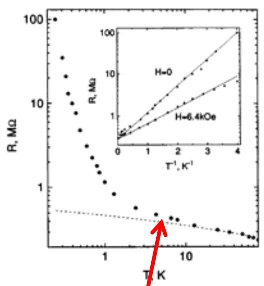
$\xi_{2D} \sim l_e e^{\frac{\pi}{2}k_F l_e}$

$$g = A_d \left(\frac{k_F W}{2\pi}\right)^{d-1}\frac{l_e}{L} \quad \sim L^{d-2}$$

$d=1 \quad (quasi-1D)$

$\Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L} \qquad \frac{\Delta g}{g} = -\frac{L_\phi}{M\, l_e}$

Localization length $\qquad \xi = M\, l_e$



$M \sim 10 \qquad l_e \sim 20nm \qquad L_\phi \sim \xi$

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M.E. Gershenson et al.

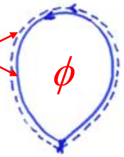
Crossover from Weak to Strong Localization in Quasi-One-Dimensional Conductors

The crossover from weak to strong localization in the resistance of quasi-1D conductors is observed for the first time with decreasing the temperature; it occurs when the phase-breaking length becomes comparable with the localization length. The signature of the strong-localization regime is an activation-type temperature dependence of the resistance and exponentially strong negative magnetoresistance. The magnetoresistance is well described by the theory of doubling of the localization length in quasi-1D conductors in strong fields; this provides a direct measurement of the localization length.

Phase coherence and magnetic field

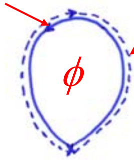
$P_{cl}(t)$ $\qquad$ $P_{int}(t)$

$2\pi\frac{\phi}{\phi_0}$



Diffuson

$2\pi\frac{\phi}{\phi_0} \qquad -2\pi\frac{\phi}{\phi_0}$



Cooperon

Cooperon: in a magnetic flux, paired trajectories get opposite phases

→ phase difference $4\pi\frac{\phi}{\phi_0}$

→ Oscillations of period $\frac{\phi_0}{2} = \frac{h}{2e}$

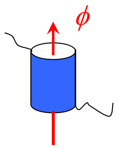
In a magnetic field, dephasing between time reversed trajectories

→The cooperon oscillates with flux

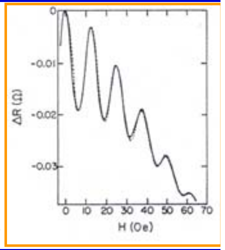
→ It cancels in a magnetic field

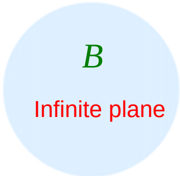
Effect of magnetic field (qualitative)

$$P_{\text{int}}(t) = P_{cl}(t) e^{4i\pi \frac{\phi}{\phi_0}}$$



Sharvin-Sharvin



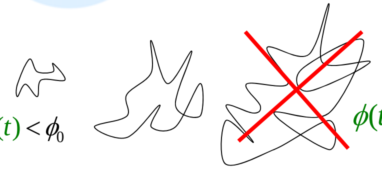


$$P_{\text{int}}(t) = P_{cl}(t) \left\langle e^{4i\pi \frac{\phi(t)}{\phi_0}} \right\rangle$$

Trajectories which enclose more than one flux quantum

do not contribute to $P_{\text{int}}(t)$

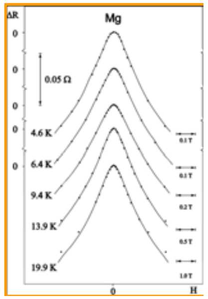
$$\sim e^{-t/\tau_B}$$



$\phi(t) < \phi_0$ $\phi(t) > \phi_0$

$$BD\tau_B = \phi_0$$

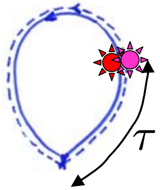
Bergman

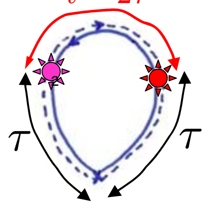


Weak-localization = phase coherence

$P_{cl}(t)$ Loop of time t $P_{\text{int}}(t)$

Diffuson (classical)





Cooperon (quantum)

Phase coherence broken after a typical time τ_ϕ
Only trajectories of time $t < \tau_\phi$ contribute to the return probablity and to the WL

$$P_{\text{int}}(t) = P_{cl}(t) e^{-t/\tau_\phi} e^{4i\pi \frac{\phi}{\phi_0}}$$

Magnetic impurities, e-e interaction, magnetic impurities
Altshuler, Aronov, Khmelnitskii

Summary

$$\Delta g = -2 \int_0^\infty P_{\text{int}}^B(t) \left(e^{-t/\tau_\phi} - e^{-t/\tau_e} \right) \frac{dt}{\tau_D}$$

Contributions of closed diffusion trajectories whose size is limited by
Size of the system, phase coherence, magnetic field, etc.

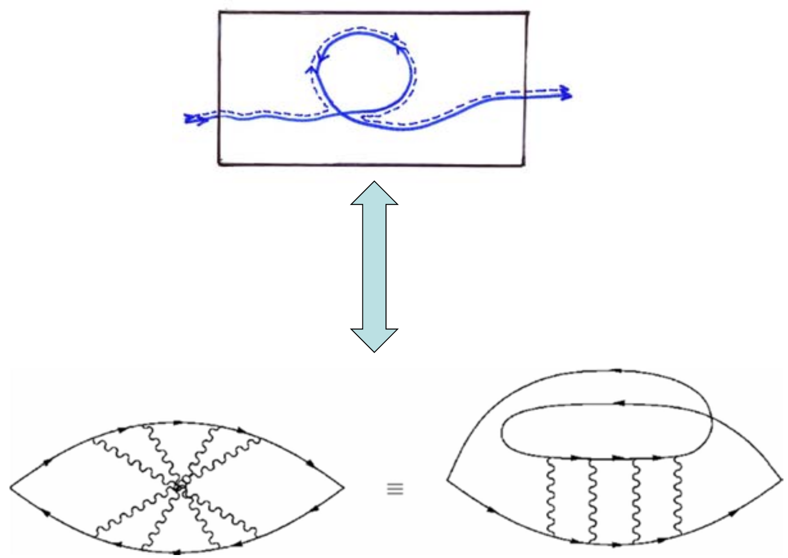
$$\Delta g \sim -2 \int_0^{\min(\tau_D, \tau_\phi, \tau_B)} \left(\frac{\tau_D}{4\pi t}\right)^{d/2} \frac{dt}{\tau_D}$$

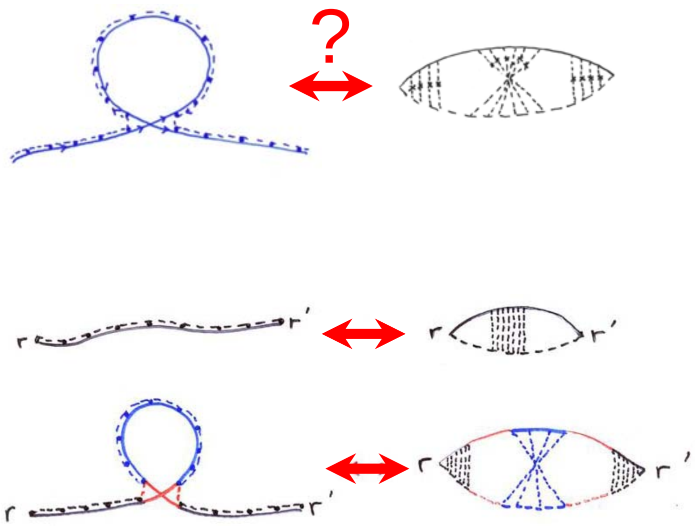
$$\tau_c \sim \min(\tau_D, \tau_\phi, \tau_B)$$

$$\Delta g = -\frac{L_c(T)}{L} \qquad d = 1 \quad (quasi-1D)$$

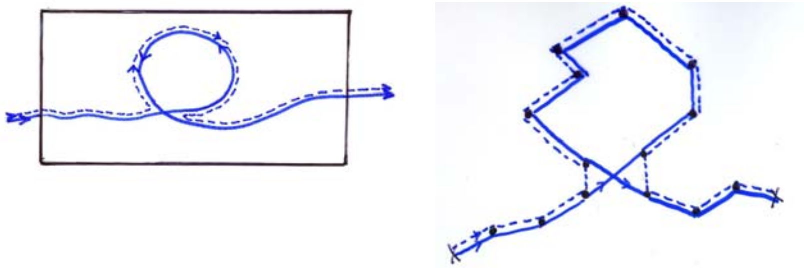
$$\Delta g = -\frac{1}{\pi} \ln \frac{L_c(T)}{l_e} \qquad d = 2$$

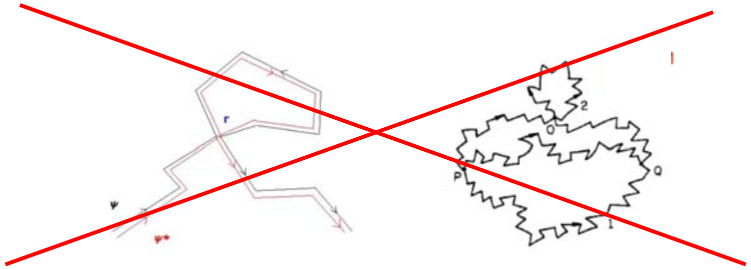
$$L_c = \sqrt{D\tau_c}$$





Quantum crossing is the reason why the WL correction is small



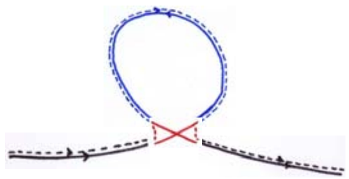


These representations are incorrect because they do not exhibit the crossing

Quantum Lego

Quantum transport of electrons and light in diffusive systems

« Lego » Classical diffusion (diffuson or cooperon)
Quantum crossings



Simple formulation of phase coherent properties in the limit g>>1